Literal and Figurative Use of Symbolism in Mathematics of Finance

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As we all know, notation and symbolism is used in all branches of Mathematics, in order to simplify, clarify and streamline the mathematical analysis. However, such notation is typically expected to be taken figuratively, rather than literally. For example, in the field of conditional probability, notation is commonly used to represent the conditional probability of occurrence of an event A, under the premise that another event B has occurred. That notation is typically expressed as P(A/B), where the diagonal symbol "/" is read as "given that…" or "assuming that…", but is NOT meant to literally represent the division of A by B.

However, we have developed similar notation in the field of Mathematics of Finance, which may be taken either figuratively or literally with equal effectiveness. It is assumed that the reader has an elementary knowledge of the fundamental concepts of Mathematics of Finance.

We wish to focus on those Mathematics of Finance formulas which we may use to help solve some fundamental questions in that field. These questions are:

(1) If we were to invest a certain amount of money **$P** NOW, at a certain constant interest rate of **i** per compounding period, but without making any withdrawals or other deposits during any of the next **n** compounding periods, what future account balance **$F** might we have, at the end of that **n** period time span ?
The answer is: **$P** times the **(F/P)** factor. We read this factor, either (figuratively) as the "**F given P**" factor , or (literally) as the "**F over P**" factor.

(2) If we were to invest a certain amount of money **$P** NOW, at a certain constant interest rate of **i** per compounding period, without making any other deposits during the next **n** compounding periods, what constant dollar amount **$A** might we be able to withdraw from the account at the end of each of the next **n** compounding periods, leading to the liquidation of that account, immediately after the **n**th withdrawal?
The answer is: **$P** times the **(A/P)** factor. We read this factor, either (figuratively) as the "**A given P**" factor, or (literally) as the "**A over P**" factor.

(3) If we were to deposit a certain amount **$A**, at the end of each of the next **n** compounding periods, in an account earning a certain constant interest rate of **i** per compounding period, without making any withdrawals during that **n** period time span, what would the accrued balance **$F** be in that account, immediately after the **n**th deposit of **$A** ?
The answer is: **$A** times the **(F/A)** factor. We read this factor, either (figuratively) as the "**F given A**" factor, or (literally) as the "**F over A**" factor.

The precise mathematical formulas which represent the responses to the questions (1), (2) and (3), respectively, are listed below:

(1) $F = P(1 + i)^n$
(2) $A = P \{ i/ [1 - (1 + i)^{-n}] \}$
(3) $F = A \{ [(1 + i)^n - 1] /i \}$

The reader is encouraged to review the book listed in the bibliography, for the details of the algebraic derivation of the above formulas.

In the use of the above formulas, we refer to:

(1) the expression $(1 + i)^n$ as the **(F/P)** factor; we also define the reciprocal of $(1 + i)^n$ as the **(P/F)** factor.

(2) the expression $\{ i/ [1 - (1 + i)^{-n}] \}$ as the **(A/P)** factor; we also define the reciprocal of $\{ i/ [1 - (1 + i)^{-n}] \}$ as the **(P/A)** factor.

(3) the expression $\{ [(1 + i)^n - 1] /i \}$ as the **(F/A)** factor; we also define the reciprocal of $\{ [(1 + i)^n - 1] /i \}$ as the **(A/F)** factor.

The "/" symbol used in the **(F/P)**, **(A/P)** and **(F/A)** factors is merely notation and is NOT intended to symbolize division. However, we will now show that the literal use of the "/" symbol is also a reasonable interpretation. That is, we will quantitatively show that the outcome of the algebraic multiplication of factors will always lead to a product which is consistent with the notion of the "/" symbol being used to represent division. We will use the symbol * to represent the operation of multiplication. We provide these algebraic proofs below:

(1) TO PROVE : (P/A) * (F/P) = (F/A)

$$
\begin{aligned}
(P/A) * (F/P) &= \{[1 - (1 + i)^{-n}]/i \} * (1 + i)^n \\
&= \{ (1 + i)^n - [(1 + i)^n (1 + i)^{-n}] \}/ i \\
&= \{ [(1 + i)^n - 1] /i \} \\
&= (F/A)
\end{aligned}
$$

(2) TO PROVE : (P/A)* (A/F) = (P/F)

$$
\begin{aligned}
(P/A) * (A/F) &= \{ [1 - (1 + i)^{-n}] /i \} * \{ i / [(1 + i)^n - 1]\} \\
&= \{ [1 - (1 + i)^{-n}] * (1 + i)^n \} / \ * \{ [(1 + i)^n - 1] * (1 + i)^n\} \\
&= \{ [(1 + i)^n - 1] \} / \{ [(1 + i)^n - 1] * (1 + i)^n\} \\
&= (1+ i)^{-n} \\
&= (P/F)
\end{aligned}
$$

(3) TO PROVE: (F/A) * (P/F) = (P/A)

$$
\begin{aligned}
(F/A) * (P/F) &= \{ [(1 + i)^n - 1] /I \} * (1 + i)^{-n} \\
&= \{[(1 + i)^n * (1 + i)^{-n}] - (1 + i)^{-n}\} / i \\
&= [1 - (1 + i)^{-n}] / I \\
&= (P/A)
\end{aligned}
$$

Bibliography

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